New Features in LS-DYNA EFG Method for Solids and Structures Analysis

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Summary:

In this presentation, an update on LS-DYAN EFG method for solids and structures analysis will be given. Several features were developed in the past two years to solve specific challenging problems as well as to improve the efficiency. This talk will emphasize on three new features including an adaptive Meshfree scheme based on a local Maximum Entropy approximation for metal forging and extrusion analysis, a semi-Lagrangain formulation in foam materials under severe compression, and a discrete meshfree approach in the failure analysis of brittle materials. Several practical examples are included to demonstrate these capabilities.
New Features in LS-DYNA EFG Method for Solids and Structures Analysis

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Current EFG Formulations for Solids and Structures Analysis

- Metal materials in Forging/Extrusion analysis: Adaptive formulation
- Foam materials: Semi-Lagrangian kernel formulation
- Rubber materials: Lagrangian kernel formulation
- Quasibrittle material fracture: Strong discontinuities formulation
- E.O.S. materials: Eulerian kernel formulation (trial version)
- Meshfree Shell: Lagrangian kernel, adaptivity …
Adaptive Methods for Manufacturing Simulations

Reasons for Adaptivity
- High accuracy requirement (surface representation, high gradient …)
- Residual stress effects the crash result

Current Numerical Limitations
- RH-adaptivity for solids (H-adaptivity is limited to shell structures).
- No failure is allowed if failure energy is important (can not be extended to metal cutting, riveting ..)
- Do not apply to rubber-like materials

1. Adaptive EFG Method

Adaptive Forging/Extrusion analysis
- An explicit/implicit solver coupled with thermal analysis.
- Introduce a fast transformation meshfree method and a modified Maximum Entropy approximation to improve the efficiency.
- A second-order interpolation scheme for state variable transfer.
- Include global/local adaptive refinements.
- Available in SMP and MPP.
EFG Fast Transformation Method

\[ \rho \dot{v} = \nabla \cdot \sigma + b \]

\[ \dot{\rho} = -\rho \nabla \cdot v \]

Local MAXENT (Ortiz and Arroyo, 2006)

\[
\begin{align*}
\text{maximize} & \quad H(p) = \beta(x) \sum_{i=1}^{N} p_i |x_i - x|^2 + \sum_{i=1}^{N} p_i \log p_i \\
\text{subject to} & \quad p_i \geq 0, i = 1, \ldots, N \\
& \quad \sum_{i=1}^{N} p_i = 1 \\
& \quad \sum_{i=1}^{N} p_i (x_i - x) = 0
\end{align*}
\]

- for \( \beta \in [0, +\infty) \), \( H(p) \) is continuous and strictly convex in solution
- well-behaved mass matrix, monotonicity, variation diminishing …
- less dependent
- difficult to decide \( \beta \)
EFG Modified Maximum Entropy Method

Define the partition function $Z: Z(x, \lambda) = \sum_{i=1}^{N} \phi_i(x) k_i^e(x, x_i) / r_i$

where $\phi_i(x)$ is the kernel function at node $i$

$r_i$ is the support size of kernel at node $i$

The unique solution of MAXENT is proven to be

$p_i(x_i, \lambda) = \frac{\phi_i(x_i) k_i^e(x_i, x)}{Z(x_i, \lambda)} \quad \forall p_i \geq 0, i = 1, \ldots, N$

satisfying

\[ \sum_{i=1}^{N} p_i = 1 \]

\[ \sum_{i=1}^{N} p_i (x_i - x) = \theta \]

where $f_i(x, \lambda) = \lambda \cdot \{(x - x_i) / r_i\}$

— implicit solve; 3~5 iterations

Mesh-free Interpolation for Data Transfer in Adaptivity

Current variable update:

$f^{n+1}_j \approx A^{n+1}_{ij} f_i = A^{n+1}_{ij} A^{n+1}_{ij} f^r_j$

$A_{ij} = \mathbf{\Psi}^{(m)}(x, t_{n+1})$

- Non-negative approximation
- Smoothness in irregular nodes
- Less dependence
- Kronecker-Delta at boundary

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Essential Boundary Conditions

Card 2

<table>
<thead>
<tr>
<th>Variable</th>
<th>DX</th>
<th>DY</th>
<th>DZ</th>
<th>ISPLINE</th>
<th>IDILA</th>
<th>IEBT</th>
<th>IDIM</th>
<th>TOLDEF</th>
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IEBT
EQ. 1: Full transformation (default)
EQ. -1: (w/o transformation)
EQ. 2: Mixed transformation
EQ. 3: Coupled FEM/EFG
EQ. 4: Fast transformation
EQ. -4: (w/o transformation)
EQ. 5: Fluid particle (trial version)
EQ. 7: Modified Maximum Entropy approximation

Input Format

*SECTION_SOLID_EFG

Forging Simulation

5827 nodes 13661 nodes
Volume change (1.7%)
Force
Global Refinement

Local Refinement

Extrusion Simulation

Comparisons of Implicit and Explicit Analysis

Force

Volume change (4.0%)
Global Refinement

Local Refinement

Extrusion with Thermal Coupling

15997 nodes

13969 nodes  15091 nodes  15003 nodes  15086 nodes
2. The Stabilized EFG Method with Kernel Switch

The Stabilized EFG Method with kernel switch

- Is a one-point integration scheme + gradient type hourglass control.
- Assumed strain method for nearly incompressible materials.
- Designed especially for foam and rubber materials.
- The speed is between FEM reduced integration element (#1) and full integration element (#2).
- A switch to full integration (rubber) or Semi-Lagrangian kernel (foam) is allowed in large deformation range.
- Available in SMP explicit and MPP explicit.

\[ \Psi^{(m)} = \Psi^{(m)}_{\text{strain}} + (x-x_0)\Psi^{(m)}_{\text{hour}}, \quad (y-y_0)\Psi^{(m)}_{\text{hour}}, \quad (z-z_0)\Psi^{(m)}_{\text{hour}} + O_2 \]

\[ \mathbf{B}^{(m)} = B^{(m)}_{\text{strain}} + (x-x_0)B^{(m)}_{\text{hour},x} + (y-y_0)B^{(m)}_{\text{hour},y} + (z-z_0)B^{(m)}_{\text{hour},z} \]

Assumed Strain Method

\[ \tilde{\mathbf{e}} = \mathbf{B}\tilde{U} \]

\[ \mathbf{B} = \mathbf{B}_0 + \mathbf{B}_x(x-x_0) + \mathbf{B}_y(y-y_0) + \mathbf{B}_z(z-z_0) \]

\[ \mathbf{B}_x = \begin{bmatrix} \frac{\partial^2}{\partial x^2} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} & \frac{\partial^2}{\partial y^2} & \frac{\partial^2}{\partial z^2} & \frac{\partial^2}{\partial x \partial y} & \frac{\partial^2}{\partial x \partial z} & \frac{\partial^2}{\partial y \partial z} \end{bmatrix} \Psi^T \]

Total Lagrangian

\[ \sigma_0 = F_{xx}S_{x} \delta_{y,z} + F_{yy}S_{y} \delta_{x,z} + F_{zz}S_{z} \delta_{y,x} + F_{xy}S_{xy} \delta_{x,y} + F_{xz}S_{xz} \delta_{x,z} + F_{yz}S_{yz} \delta_{y,z} \]
Convective velocity $C$ due to Semi-Lagrangian or Eulerian kernel

\[ v = \frac{\partial X(t_{n+1})}{\partial t} \quad \text{(material time frame)} \quad C = v - v' \]

\[ v = \frac{\partial X(t_{n+1})}{\partial t} \quad \text{(reference time frame)} \quad f = \frac{\partial f}{\partial t} + (C \cdot \nabla)f \]

- Lagrangian phase: $\Psi^{[n]}(x_{n}, t_{n}) = \Psi^{[n]}$
- Transport phase: $\partial \Psi^{[n]}/\partial t + (C \cdot \nabla)f = 0$

\[ \nabla f' = \sum_{\lambda} \frac{\partial \Psi^{[n]}_{\lambda}}{\partial x} f_{\lambda}; f_{\lambda} = \sum_{\lambda} \nabla \Psi^{[n]}_{\lambda}(x_{\lambda}, t_{n}) \partial \Psi^{[n]}_{\lambda}/\partial x \]

Stress recovery scheme is conservative, consistent and monotonic!

### Input Format

**SECTION_SOLID_EFG**

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- **IEBT**
  - EQ. 1: Full transformation (default)
  - EQ.-1: (w/o transformation)
  - EQ. 2: Mixed transformation
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  - EQ. 4: Fast transformation
  - EQ. 4: (w/o transformation)
  - EQ. 5: Fluid particle (trial version)
  - EQ. 7: Modified Maximum Entropy approximation

- **IDIM**
  - EQ. 1: Local boundary condition method (default)
  - EQ. 2: Two-points Gauss integration
  - EQ.-1: Stabilized EFG method
Deformation tolerance for the activation of Semi-Lagrangian kernel

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TOLDEF = 0.0 : Lagrangian kernel
TOLDEF > 0.0 : Semi-Lagrangian kernel
TOLDEF < 0.0 : Eulerian kernel

Time control for the activation of Semi-Lagrangian kernel or Eulerian kernel

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Rubber Bushing Analysis using Stabilized EFG Method

Mooney-Rivlin Rubber
Poisson’s = 0.4995
Stabilized EFG explicit analysis
Switched to full integration at \( t=100 \)
Completion at \( t=150 \)

CPU comparison at \( t=50 \)

<table>
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<tr>
<th>Methods</th>
<th>S-FEM(#1)</th>
<th>F-FEM(#2)</th>
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Foam Compression using Stabilized EFG Method and Semi-Lagrangian Kernel

Low Density Foam
Stabilized EFG explicit analysis
Switched to Semi-Lagrangian (TOLDEF=0.01)

Original EFG

EFG + Semi-Lagrangian Kernel

Comparison of final deformation

3. EFG Failure Analysis

Meshfree Failure Analysis

- Is a discrete approach.
- Crack initiation and propagation are governed by cohesive law.
- Crack currently is cell-by-cell propagation and is defined by visibility.
- Minimized mesh sensitivity and orientation effects.
- Applied to quasibrittle materials.
Discrete Cracks

**Crack in Meshfree**: Visibility Criterion (Belytschko et al. 1996)

Intrinsic (Implicit crack): no additional unknowns

\[
\begin{align*}
\Omega_0 & \quad \rightarrow \quad \phi_j^{\text{MFS}}(\xi) \rightarrow x(\eta) = \sum_{j=1}^k \phi_j^{\text{MFS}}(\eta) x_j + \frac{1}{2} \left( \sum_{j=1}^k \nabla \phi_j^{\text{MFS}}(\eta) \right) \left( \nabla x_j \right) \\
\Omega_\eta & \quad \rightarrow \quad \frac{\partial x(\eta)}{\partial \eta} = \sum_{j=1}^k \phi_j^{\text{MFS}}(\eta) \left( \frac{\partial \nabla \phi_j^{\text{MFS}}(\eta)}{\partial \eta} \right) + \frac{1}{2} \left( \sum_{j=1}^k \nabla \phi_j^{\text{MFS}}(\eta) \right) \left( \frac{\partial \nabla \phi_j^{\text{MFS}}(\eta)}{\partial \eta} \right) \left( \partial x_j \right)
\end{align*}
\]

Initially-rigid Cohesive Law: Redefined Displacement Jump (Sam, Papoulia and Vavasis 2005)

\[
\lambda = \left( \frac{u_j}{\delta_{\text{in}} + \delta_j} \right)^2 + \beta^2 \left( \frac{u_j}{\delta_{\text{in}} + \delta_j} \right)^2
\]

\[
T_0 = \sqrt{r^2 + \frac{\beta^2}{\alpha} \lambda^2} = T_{\text{max}}
\]

\[
T_e = \frac{1 - \lambda}{\lambda} u_j T_{\text{max}} \quad \text{and} \quad T_i = \frac{1 - \lambda}{\lambda} u_j \alpha T_{\text{max}}
\]

---

Minimization of Mesh Size Effect in Mode-I Failure Test

Failure is limited in this area

Coarse elements | Fine elements

\[
D = 0.01 \quad \text{and} \quad D = 0.005
\]

\[
\begin{align*}
\text{Failure Stress} & \quad \text{Time} \quad \text{Load} \quad \text{Stress} \\
\text{Min Test (MM005)} & \quad \text{Min Test (MM005)}
\end{align*}
\]
**EFG 3D Edge-cracked Plate under Loading**

- 101 x 31 x 6 nodes
- Elastic
- EFG Fracture
- Linear Cohesive Law
- Explicit analysis

**Resultant Displacement Contour**

**EFG Glass under Impact**

- 101 x 101 x 4 nodes
- Elastic + Rubber
- EFG Fracture
- Linear Cohesive Law
- Explicit analysis

**Failure Contour**
Conclusions

• Adaptive method is attractive in metal forging/extrusion simulation.

• Stabilized method designed for foam and rubber materials can be used to improve the efficiency in explicit analysis.

• Failure analysis using cohesive model and visibility can be applied to brittle and semi-brittle materials.

• Strong discontinuities formulation including XFEM will be our next focus for a more general failure analysis.